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DUALITY AND ASYMPTOTIC GEOMETRIES

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ABSTRACT

We consider a series of duality transformations that leads to a constant shift in the harmonic functions appearing in the description of a configuration of branes. This way, for several intersections of branes, we can relate the original brane configuration which is asymptotically flat to a geometry of the type $adS_k \times E^l \times S^m$. The implications of our results for supersymmetry enhancement, M(atric) theory at finite N, and for supergravity theories in diverse dimensions are discussed.

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Duality symmetries are one of the cornerstones of the second superstring revolution[1]. Using a web of duality symmetries one can connect all known string theories and eleven dimensional supergravity, leading to a conjectural master theory that contains all others as special limits (for reviews see [2]). A candidate for this master theory, the M(atrix) theory, has been recently proposed in [3]. Geometry and/or topology change of spacetime due to dualities is an extensively studied subject. A less well studied facet of this topic is the change of asymptotic geometry due to dualities. Some examples of this type have been studied before in [4, 5].

We shall show in this article using a set of duality transformations and coordinate transformations that one can map supersymmetric configurations of M -branes and D -branes that are asymptotically flat to configurations that are of the type $adS_k \times E^l \times S^m$, where adS_k is the k -dimensional anti-de Sitter space, E^l is the l -dimensional Euclidean space and S^m is the m -dimensional sphere. As we shall see, one can use a specific combination of duality and coordinate transformations that we shall call the shift transformation in order to change the constant part of the harmonic functions appearing in the D and M -brane solutions.

These results have a variety of applications. First of all they lead to a better understanding of the issue of supersymmetry enhancement near the horizon of certain black hole configurations. In addition, using our results, we shall be able to relate supergravity computations in the standard space-like compactification of M -theory to finite N M(atrix) theory. Also, our results imply that certain spontaneous compactifications of 10 and 11 dimensional supergravity should exist. In many cases, one can check that such compactifications indeed appear in the supergravity literature.

To illustrate the mechanism that converts an asymptotically flat space to an anti-de Sitter space we will start by considering the Dabholkar-Harvey solution of $N=1$ 10d supergravity[6] that describes a fundamental string. We will then generalize this result to a non-extremal string and afterwards to an arbitrary supersymmetric configuration of orthogonally intersecting branes with at least three transverse directions.

The metric, antisymmetric tensor and dilaton field of the solution that describes a fundamental string are given by the following expressions

$$\begin{aligned} ds^2 &= H(r)^{-1}(-dt^2 + dx_1^2) + (dx_2^2 + \cdots + dx_9^2), \\ B_{01} &= H(r)^{-1} - 1; \quad e^{-2\phi} = H(r) \\ H(r) &= 1 + \frac{Q}{r^6}; \quad r^2 = x_2^2 + \cdots + x_9^2, \end{aligned} \tag{1}$$

H is a harmonic function that depends only on the transverse directions.

Let us perform a T -duality in the x_1 direction. Using Buscher's rules one obtains

$$\begin{aligned} ds^2 &= (H - 2)dt^2 + Hdx_1^2 + 2(1 - H)dtdx_1 + (dx_2^2 + \cdots + dx_9^2), \\ B &= 0; \quad e^{-2\phi} = 1 \end{aligned} \tag{2}$$

We now make a change of coordinates that amounts to an $SL(2, R)$ transformation

$$\begin{pmatrix} t \\ x_1 \end{pmatrix} = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} t' \\ x'_1 \end{pmatrix}; \quad \alpha\delta - \beta\gamma = 1, \quad \beta \neq \delta. \quad (3)$$

Finally, we perform another T -duality transformation in the x'_1 direction⁴. The result is (1), but with the harmonic function replaced by

$$H \rightarrow (\beta - \delta)((\beta - \delta)H - 2\beta) \quad (4)$$

and, in addition, the constant part of B_{01} is now equal to $(\alpha - \gamma)/(\beta - \delta)$, which can be shifted to 1 by a constant gauge transformation (for generic $SL(2, R)$ transformations there may be global obstructions to these gauge transformations; these are absent for the cases considered below). It follows that if one chooses the $SL(2, R)$ with $\delta = -\beta$ the resulting configuration has harmonic function with zero constant part and therefore different asymptotics. If in addition we demand that the charge \mathcal{Q} remains unchanged then this fixes the parameters (up to an overall sign) to be $\delta = -\beta = 1/2$. This still leaves a one-parameter family of $SL(2, R)$ transformations, given by $\alpha + \gamma = 2$, that yields the same result. There is, however, a global issue that we have not yet discussed. In order to perform the T -duality in the x_1 direction we need to compactify this direction. Assuming that t is non-compact, (t, x_1) have the geometry of a cylinder. Generically, an $SL(2, R)$ transformation is not well-defined on a cylinder. However, there is a member in the one-parameter family of $SL(2, R)$ transformations given above that is well-defined. This is the case for $\gamma = 0, \alpha = 2$. In this case,

$$t' = \frac{1}{2}(t + x_1) \quad x'_1 = 2x_1. \quad (5)$$

Notice that both T -dualities that are involved in the shift transformations are along a space-like direction. A similar set of transformations (with $\gamma = 0$) has been used by Hyun[8] in order to connect $4d$ and $5d$ black holes to the three-dimensional BTZ black hole[9]. We also note that the above family of $SL(2, R)$ transformations contains the case, $\alpha = \gamma = 1$, which brings one from (t, x_1) to the light-cone frame. Finally, notice that the transformations just described can be thought of as part of an $O(2, 2)$ T -duality group associated with the isometries in the (t, x_1) directions.

Let us generalize this result to non-extremal solutions. A simple algorithm that leads to a non-extreme version of a given extreme solution has been given in [10]. The mechanism that we just described for the extremal case generalizes straightforwardly to the non-extremal case. To illustrate the method we shall discuss the non-extreme string solution of [11] (which can also be viewed as a double dimensional reduction of the non-extremal $11d$ membrane solution of Güven[12]). The metric, antisymmetric tensor and dilaton field are given by

$$\begin{aligned} ds^2 &= H(r)^{-1}(-f(r)dt^2 + dx_1^2) + f(r)^{-1}dr^2 + r^2 d\Omega_7^2, \\ B_{01} &= H'(r)^{-1} - 1, \quad e^{-2\phi} = H(r), \quad r^2 = x_2^2 + \cdots + x_9^2 \end{aligned} \quad (6)$$

⁴ T -duality along a linear combination of space-like coordinates has been considered in [7].

where

$$\begin{aligned} H'(r)^{-1} &= 1 - \frac{Q}{r^6} H^{-1}; \quad Q = \mu \sinh \psi \cosh \psi; \\ H(r) &= 1 + \frac{Q}{r^6}; \quad Q = \mu \sinh^2 \psi; \\ f(r) &= 1 - \frac{\mu}{r^6} \end{aligned} \tag{7}$$

The extreme solution is found in the limit $\mu \rightarrow 0$, $\psi \rightarrow \infty$, while the charge Q is kept fixed. In this limit one gets $H' = H$, $Q = Q$, $f = 1$, and the solution (6) collapses to the extreme one in (1).

One can now perform the set of transformations $T_1 SL(2, R) T_1$ (we use the notation T_i to denote the T -duality transformation in the x_i direction) exactly in the same way as for the extremal case. The result is that one can remove the constant part of the harmonic function H provided that $\delta = -\beta = (1 + \coth \psi)^{-1}$, where the last equality is required only if one wants to preserve the values of the charges. In addition, B_{01} is transformed (again up to constant gauge transformation) to $B_{01} \rightarrow (H - 1)^{-1} - 1$. The parameters α and γ are still restricted by the determinant condition, $\alpha\delta - \beta\gamma = 1$. A similar set of transformations (with $\gamma = 0$) has been considered previously in [8]. One easily checks that the values of β, δ coincide with the extremal ones in the extremal limit.

We now show that a configuration consisting of four or fewer fundamental strings, solitonic fivebranes, D -branes, waves and Kaluza-Klein monopoles in type II theory, subject to the conditions below, can be mapped by means of a series of duality transformations to a specific configuration of four (or fewer) $D3$ -branes. It is then shown that the harmonic functions in the resulting configuration can be shifted in a similar fashion as shown above. We assume throughout that all harmonic functions depend only on overall transverse directions and that there are at least three of the latter, so that the harmonic functions are bounded at infinity. Furthermore, we require the configurations to be built according to the intersection rules based on the ‘no-force’ condition ([13] and references therein). Together, these requirements imply that there are at most four independent charges. Moreover, the fraction of supersymmetry preserved is $1/2^n$ for $n \leq 3$ and $1/8$ for $n = 4$ if n is the number of charges.

Starting with an arbitrary set of branes, waves and monopoles in type IIA theory, subject to the above conditions, the allowed configurations contain at most one wave and one monopole[14]. By making a T -duality transformation these can be mapped into a fundamental string and a solitonic fivebrane, respectively. The configuration with four charges (the case with fewer charges can be dealt with similarly) can now be related to four $D3$ -branes in type IIB: each brane can be turned into a D -brane using S -duality, and can then be mapped into a $D3$ -brane using T -duality. At each step in this procedure there is enough freedom to choose the directions along which to T -dualize to make sure that the $D3$ -branes already present remain $D3$ -branes (which are then inert under S -duality), and to avoid mapping any of the other branes into a wave or a

monopole.

In order to show that the harmonic functions in the resulting configuration of four $D3$ -branes can each be shifted by a constant term, let us choose coordinates such that the branes are in the (x_1, x_2, x_3) , (x_1, x_4, x_5) , (x_2, x_4, x_6) and (x_3, x_5, x_6) directions, respectively; there are three overall transverse coordinates x_7, x_8, x_9 . We now make a T_{23} and S transformation to obtain a fundamental string, a solitonic fivebrane, and two $D3$ -branes described by the following background fields

$$\begin{aligned}
ds^2 &= H_1^{-1} H_3^{-\frac{1}{2}} H_4^{-\frac{1}{2}} (-dt^2) + H_1^{-1} H_3^{\frac{1}{2}} H_4^{\frac{1}{2}} (dx_1^2) \\
&\quad + H_3^{\frac{1}{2}} H_4^{-\frac{1}{2}} (dx_2^2 + dx_5^2) + H_3^{-\frac{1}{2}} H_4^{\frac{1}{2}} (dx_3^2 + dx_4^2) + H_2 H_3^{-\frac{1}{2}} H_4^{-\frac{1}{2}} (dx_6^2) \\
&\quad + H_2 H_3^{\frac{1}{2}} H_4^{\frac{1}{2}} (dx_7^2 + dx_8^2 + dx_9^2) \\
B_{01} &= H_1^{-1} - 1 \quad H_{ijk} = (dB)_{ijk} = \epsilon_{ijkl} \partial_l H_2 \quad (i, j, k, l = 6, 7, 8, 9) \\
A_{0346} &= -H_3^{-1} \quad A_{0256} = -H_4^{-1} \\
e^{-2\phi} &= H_1 H_2^{-1}
\end{aligned} \tag{8}$$

The H_i are harmonic functions, depending only on the overall transverse directions. With this configuration we can again perform the same steps as before, i.e. make a T -duality transformation in the x_1 direction, followed by a change of coordinates, and then again a T -duality transformation in the x_1 direction. Because this direction is along the solitonic fivebrane, and orthogonal to both D -branes, the corresponding harmonic functions appear with the appropriate powers for this procedure to result in just changing the harmonic function H_1 by a constant shift as before. We can now return to the initial configuration by taking the $T_{23}S$ -dual. As this configuration is completely symmetric in the four $D3$ -branes, we can similarly shift any other harmonic function by a constant.

The shift transformations in the harmonic functions are particularly interesting for the case of brane configurations with some simple near horizon geometry. Such branes interpolate between this geometry and Minkowski spacetime at infinity. In eleven dimensions, the membrane ($M2$) interpolates between \mathcal{M}_{11} and $adS_4 \times S^7$, whereas the fivebrane ($M5$) interpolates between \mathcal{M}_{11} and $adS_7 \times S^4$ [15]. We may dimensionally reduce the $M2$ -brane to a fundamental (type IIA) string, do the shift procedure as explained before, and lift this configuration up to eleven dimensions again. This will give the $adS_4 \times S^7$ solution of eleven-dimensional supergravity⁵, which is then of course just the $M2$ -brane solution with the harmonic function not containing the constant term. For the $M5$ -brane one can similarly show that it is connected via dualities and a simple coordinate transformation to the $adS_7 \times S^4$ geometry. In fact, $adS_4 \times S^7$ and $adS_7 \times S^4$ are known to be maximally supersymmetric vacua of $d=11$ supergravity. There is one ten-dimensional p -brane with similar properties: the self-dual threebrane. Its near horizon geometry, $adS_5 \times S^5$, is itself a maximally supersymmetric vacuum of type IIB supergravity. All this raises the question,

⁵This corresponds to the Freund-Rubin compactification [16] to four dimensions with maximal supersymmetry.

however, how two such solutions can be related by symmetry transformations, as one of them (the brane) breaks half the supersymmetry and the other is maximally supersymmetric. We get back to this paradox after we have discussed supersymmetry enhancement.

Let us consider orthogonal M -brane intersections with a simple near horizon geometry which is a product containing an anti-de Sitter spacetime and a sphere. There is only one intersection of two M -branes which falls into this class, namely the $M2 \perp M5$ solution,

$$\begin{aligned} ds^2 &= H_2^{-\frac{2}{3}} H_5^{-\frac{1}{3}} (-dt^2 + dx_1^2) + H_2^{-\frac{2}{3}} H_5^{\frac{2}{3}} (dx_2^2) + H_2^{\frac{1}{3}} H_5^{-\frac{1}{3}} (dx_3^2 + \cdots + dx_6^2) \\ &\quad + H_2^{\frac{1}{3}} H_5^{\frac{2}{3}} (dx_7^2 + \cdots + dx_{10}^2), \\ F_{r012} &= \pm \partial_r H_2^{-1}, \quad F_{2\alpha\beta\gamma} = \pm \epsilon_{\alpha\beta\gamma} \partial_r H_5, \end{aligned} \quad (9)$$

where the signs differentiate between brane and anti-brane, and $\epsilon_{\alpha\beta\gamma}$ is the volume form of the three-sphere surrounding the intersection. The harmonic functions are

$$H_2 = 1 + \frac{Q_2}{r^2}, \quad H_5 = 1 + \frac{Q_5}{r^2}, \quad (10)$$

where $r^2 = x_7^2 + x_8^2 + x_9^2 + x_{10}^2$. With this choice of harmonic functions the solution is asymptotically Minkowski. The near horizon geometry is the geometry for $r \rightarrow 0$. In this limit the constant parts of the harmonic functions become negligible. Of course, (9) with harmonic functions without the constant term is still a solution of the supergravity. The corresponding geometry can also be obtained throughout spacetime by performing the shift transformation. We get

$$\begin{aligned} ds^2 &= Q_2^{-\frac{2}{3}} Q_5^{-\frac{1}{3}} r^2 (-dt^2 + dx_1^2) + Q_2^{-\frac{2}{3}} Q_5^{\frac{2}{3}} (dx_2^2) + Q_2^{\frac{1}{3}} Q_5^{-\frac{1}{3}} (dx_3^2 + \cdots + dx_6^2) \\ &\quad + Q_2^{\frac{1}{3}} Q_5^{\frac{2}{3}} \left(\frac{1}{r^2} dr^2 + d\Omega_3^2 \right). \end{aligned} \quad (11)$$

Thus the spacetime factorizes into the product of an adS_3 spacetime (with coordinates t, x_1, r), a three-sphere S^3 of radius $Q_2^{\frac{1}{6}} Q_5^{\frac{1}{3}}$, and a flat Euclidean five-dimensional space E^5 . Similarly, one finds the other possible orthogonal intersections that give rise to this kind of geometry: for three charges these are the $M2 \perp M2 \perp M2$ and $M5 \perp M5 \perp M5$ (with three overall transverse directions), and for four charges the $M2 \perp M2 \perp M5 \perp M5$ intersection. Table 1 shows the corresponding geometries. These near horizon geometries have also been given in [17]. Being solutions to the field equations, the geometries in table 1 can also be interpreted as spontaneous compactifications of eleven-dimensional supergravity on a sphere and/or a torus (after periodically identifying Euclidean coordinates).

$M2 \perp M5$	$adS_3 \times E^5 \times S^3$
$M2 \perp M2 \perp M2$	$adS_2 \times E^6 \times S^3$
$M5 \perp M5 \perp M5$	$adS_3 \times E^6 \times S^2$
$M2 \perp M2 \perp M5 \perp M5$	$adS_2 \times E^7 \times S^2$

Table 1

The product geometries have the form $adS_{p+2} \times E^q \times S^{9-p-q}$, where p is the spatial dimension of the intersection, q is the number of relative transverse coordinates and $9 - p - q$ is the number of overall transverse coordinates minus one. A wave can be added to the common string of the $M2 \perp M5$ and $M5 \perp M5 \perp M5$ intersections (as well as to the single $M2$ and $M5$ -branes). It modifies only the adS_3 part of the corresponding geometry. After reduction to ten dimensions, the $M2 \perp M5$ intersection with a wave is dual to the $M2 \perp M2 \perp M2$ intersection, and the $M5 \perp M5 \perp M5$ intersection with a wave is dual to the $M2 \perp M2 \perp M5 \perp M5$ intersection.

It is well-known[18] that some solutions exhibit supersymmetry enhancement at the horizon. For example, the $M2$ and $M5$ -branes break one half of supersymmetry, whereas their near horizon geometries $adS_4 \times S^7$ and $adS_7 \times S^4$ are maximally supersymmetric vacua of $d = 11$ supergravity. A number of other cases of supersymmetry enhancement for p -branes in different dimensions is known, and in all these cases the near horizon geometry contains a factor $adS_k \times S^m$. The self-dual threebrane in ten dimensions is another example. We find that in fact all intersections of table 1 exhibit supersymmetry enhancement at the horizon. This becomes clear by the observation that the $adS_{p+2} \times E^q \times S^{9-p-q}$ solutions themselves preserve twice as much supersymmetry as the corresponding M -brane intersections. We illustrate this for the $adS_3 \times E^5 \times S^3$ solution corresponding to the $M2 \perp M5$ intersection.

Unbroken supersymmetries correspond to vanishing supersymmetry variations of the gravitino,

$$\delta\psi_M = D_M\varepsilon + \frac{1}{288}(\Gamma_M^{NPQR} - 8\delta_M^N\Gamma^{PQR})F_{NPQR}\varepsilon = 0, \quad (12)$$

in the background (9), with $H_2 = \frac{1}{r^2}$ and $H_5 = \frac{1}{r^2}$ ⁶. We split this equation in the three parts corresponding to adS_3 (with coordinates x^μ), E^5 (coordinates y^s) and S^3 (coordinates z^α). The Γ -matrices can be conveniently chosen to be

$$\begin{aligned} \Gamma_\mu &= \gamma_\mu \otimes \mathbf{1}_4 \otimes \mathbf{1}_2 \otimes \sigma_1 \\ \Gamma_s &= \mathbf{1}_2 \otimes \gamma_s \otimes \mathbf{1}_2 \otimes \sigma_3 \\ \Gamma_\alpha &= \mathbf{1}_2 \otimes \mathbf{1}_4 \otimes \gamma_\alpha \otimes \sigma_2. \end{aligned} \quad (13)$$

Substituting in (12) one finds for the E^5 components:

$$\delta\psi_s = \partial_s\varepsilon + \frac{1}{6}\mathbf{1} \otimes (-\gamma_s\gamma^2 + 3\delta_s^2) \otimes \mathbf{1} \otimes (\sigma_1 + i\sigma_2)\varepsilon = 0. \quad (14)$$

Writing $\varepsilon = \eta(x) \otimes \xi(y) \otimes \rho(z) \otimes \chi$, we see that (14) reduces to $\partial_s\xi = 0$ if we assume that the two-component spinor χ is projected onto its upper component. Thus, ξ are the constant Killing spinors of E^5 . For the adS_3 components we get

$$D_\mu\varepsilon - \frac{1}{6}\gamma_\mu \otimes \gamma^2 \otimes \mathbf{1} \otimes (\mathbf{1} + 2\sigma_3)\varepsilon = 0, \quad (15)$$

⁶Without loss of generality, we take $Q_2 = Q_5 = 1$.

and this yields

$$D_\mu \eta \mp \frac{1}{2} \gamma_\mu \eta = 0 \quad \text{for} \quad \gamma^2 \xi = \pm \xi \quad \text{and} \quad \sigma_3 \chi = \chi, \quad (16)$$

which coincides with the geometric Killing spinor equation for adS_3 . Finally, the S^3 components give

$$D_\alpha \varepsilon - \frac{i}{6} \mathbf{1} \otimes \gamma^2 \otimes \gamma_\alpha \otimes (\mathbf{1} + 2\sigma_3) \varepsilon = 0. \quad (17)$$

This translates into the geometric Killing spinor equation on S^3 :

$$D_\alpha \rho \mp \frac{i}{2} \gamma_\alpha \rho = 0 \quad \text{for} \quad \gamma^2 \xi = \pm \xi \quad \text{and} \quad \sigma_3 \chi = \chi. \quad (18)$$

Since adS_3 , E^5 and S^3 all admit the maximal number of Killing spinors and there is only one projection on χ , we conclude that this solution preserves one half of supersymmetry which is double the amount preserved by the $M2 \perp M5$ intersection in the bulk. For the other cases of table 1, we find that (12) in those backgrounds reduces to the appropriate geometric Killing spinor equations up to two projections, thus establishing supersymmetry doubling also in these cases⁷.

All known solutions that exhibit supersymmetry enhancement not only have a near horizon geometry of the form $adS_{p+2} \times E^q \times S^{d-p-q-2}$, but also have regular (i.e. finite) dilaton (if any) near the horizon. This is true for the four and five-dimensional extremal black holes with nonzero entropy, and also for the $D3$ -brane which doesn't couple to the dilaton. We observe that, for the eleven-dimensional configurations of table 1, a dimensional reduction over one or more of the relative transverse directions will always give rise to a dilaton in lower dimensions that is regular at the horizon. That is because the dilaton originates from the eleven-dimensional metric according to

$$ds_{11}^2 = e^{2\alpha\phi} ds_{10}^2 + e^{2\beta\phi} dy^2, \quad (19)$$

where y is the coordinate that is reduced over, and α and β are constants related to the choice of metric frame and the normalization of the dilaton. Since the metric diagonal component for a relative transverse direction is regular for the configurations of table 1, $e^{2\beta\phi}$ and thus ϕ is regular. Reduction over such directions also does not interfere with supersymmetry. Hence all solutions that can be obtained in this way must exhibit supersymmetry enhancement. The same is true for configurations obtained by further T -dualities in the relative transverse directions.

We now turn to the paradox raised by the observation that the shift transformation relates solutions with different amounts of unbroken supersymmetry. For clarity, we take the example of the self-dual threebrane. It is related via

⁷For the $M2 \perp M2 \perp M5 \perp M5$ configuration, there is enhancement only if the four signs related to brane/anti-brane choice multiply to -1 (in our conventions); for opposite orientations supersymmetry is completely broken.

a series of dualities plus a simple coordinate transformation to the $adS_5 \times S^5$ solution of type IIB supergravity. The latter has maximal supersymmetry, with Killing spinors that are the direct product of the Killing spinors of adS_5 and S^5 . However, in order to do the shift transformation we have to perform T -duality along some coordinates of adS_5 . To this end we have to compactify these coordinates. Only half of the Killing spinors of adS_5 spacetime survive compactification as one can immediately verify by looking at the explicit expression of the Killing spinors[19]. In other words, adS_5 with a set of compact isometries (needed in order to perform T -dualities) admits only half of the Killing spinors compared to adS_5 with no compact isometries. Actually, this is true for all anti-de Sitter spacetimes. A familiar case of this sort is that of the BTZ black hole[9]. This three-dimensional black hole can be obtained by compactifying one of the coordinates of adS_3 . The compactification kills all Killing spinors in the non-extremal case, and half of them in the massless extremal one[20]. The rotating extremal case preserves only 1/4 of the supersymmetry as it also contains a wave. A similar situation was also discussed in [19], where it is shown that a maximally supersymmetric adS_d solution can be reduced to a half-supersymmetric domain wall solution in $d-1$ dimensions. Let us note here that the case of dualization along a rotational isometry is different from the discussion above. In spaces with rotational isometries the Killing spinors also depend on the coordinates along which one dualizes. Although it appears that the spacetime supersymmetry is lost after the T -duality[5, 21], it is actually non-locally realized[22]. In our case, the T -duality maps between two configurations with the same amount of spacetime supersymmetry. Only after decompactification one can detect the enhanced supersymmetry. This resolves the paradox as the threebrane is a half-supersymmetric solution. On the other hand, dimensional reduction or T -duality along toroidal directions does not break any supersymmetry. So solutions obtained by dimensional reduction along relative transverse directions of the M -brane intersections of table 1 should still exhibit supersymmetry enhancement.

As an example, let us consider the six-dimensional dyonic string solutions of [23],

$$\begin{aligned} ds^2 &= H_2^{-1}(-dt^2 + dx_1^2) + H_5(dx_7^2 + \cdots + dx_{10}^2) \\ e^{-2\phi} &= H_2 H_5^{-1} \\ H_{r01} &= \pm \partial_r H_2^{-1} \ , \quad H_{\alpha\beta\gamma} = \pm \epsilon_{\alpha\beta\gamma} \partial_r H_5 \ , \end{aligned} \tag{20}$$

written in the six-dimensional string frame. As suggested by the way of presenting, these dyonic strings are obtained by double dimensional reductions over the relative transverse directions of the $M2 \perp M5$ intersection. If electric and magnetic charges are chosen equal ($Q_2 = Q_5$ in (10)), (20) becomes the self-dual string[24]. The solutions (20) preserve 1/4 of supersymmetry, but now we know that this must be enhanced to 1/2 at the horizon. The near horizon geometry of the strings is $adS_3 \times S^3$. The self-dual string can also be embedded into six-dimensional $N=2$ chiral supergravity where it breaks only 1/2 of supersymmetry, and hence $adS_3 \times S^3$ is a maximally supersymmetric vacuum.

We would now like to discuss the relation between this shift transformation and results recently obtained in M(atr)ix theory. Following the proposal of [3] one needs to consider M -theory in the infinite momentum frame to obtain a formulation in terms of matrix model quantum mechanics in the large N limit. Compactifying along a null-direction is conjectured to lead, in the framework of DLCQ, to a correspondence between M -theory and the Matrix model for finite N [25]. In [26] scattering amplitudes for configurations of D -branes and gravitons have been computed, both in the effective supergravity theories and through the Matrix model approach. It was observed that the supergravity calculation always produces identical results to the Matrix theory in the infinite N limit, in which subleading corrections are suppressed. However, only for the supergravity corresponding to reduction of the $11d$ theory along a light-like coordinate, the computations also agree for finite N . In this case, the harmonic functions of the $D0$ -branes in the supergravity calculation are shifted to vanish at infinity. Comparing with our results, we have found a one-parameter family of ‘frames’ in which this shift occurs, and which therefore also lead to agreement with the Matrix model computation for finite N . With ‘frames’ we mean the frames that are obtained after applying the shift transformations in the Infinite Momentum Frame. This one-parameter family of frames contains the $10d$ light-cone frame. However, it is not clear to us whether this is related to the light-cone frame in the DLCQ. In all frames considered in (3), except for (5), the shift transformation can only be applied in the supergravity limit. This is due to global issues we already discussed. For the transformation (5) such problems are absent since we do not need to compactify the time coordinate. Furthermore, the T -dualities are along space-like directions, and this transformation should be well-defined also in the full theory. Finally, let us note that in order to perform the shift transformation to a $D0$ -brane we need to assume that the background admits at least one spatial isometry.

We conclude by making some comments on the implications of our results for supergravity theories in various dimensions. Each configuration in 11 dimensions with effective geometry of the type $adS_k \times E^l \times S^m$ corresponds to a solution of $11d$ supergravity with the appropriate amount of supersymmetry. It also follows directly that, after reduction along $p \leq l$ of the flat directions, the geometry $adS_k \times E^{l-p} \times S^m$ is a solution with the same amount of supersymmetry (counting the number of spinor components) in $11 - p$ dimensions. In addition we can deduce the existence of solutions with a certain amount of supersymmetry after spontaneous compactification on the sphere S^m . These compactifications are expected to give rise to solutions of gauged supergravities in $11 - m - p$ dimensions with geometry $adS_k \times E^{l-p}$. Several of these results are well-known, such as the spontaneous compactification of $11d$ supergravity on S^7 , giving gauged $N=8$ supergravity in $d=4$, and the $adS_7 \times S^4$ and $adS_5 \times S^5$ solutions of $d=11$ supergravity and type IIB supergravity. The anti-de Sitter parts of these solutions are maximally supersymmetric vacua of gauged maximal supergravities in seven and five dimensions [27].

Let us also make some remarks about the $10d$ case. As shown in [15], the solitonic fivebrane gives rise (in the string frame) to an asymptotic geometry of

the type $\mathcal{M}_7 \times S^3$, whereas the fundamental string yields (in the solitonic five-brane metric) an asymptotic geometry of $adS_3 \times S^7$ [28]. Similarly, we find that for the Dp -branes, in the ‘dual Dp -metric’ – the metric in which the curvature and the $(8-p)$ -form field strength appear in the action with the same power of the dilaton – the asymptotic geometry is of the type $adS_{p+2} \times S^{8-p}$ (except for the $D5$ -brane, which yields again $\mathcal{M}_7 \times S^3$). As before, these metrics can not only be realised asymptotically, but everywhere in spacetime after making the shift transformation. With the exception of $adS_5 \times S^5$ these solutions break $1/2$ of supersymmetry. We note that the $\mathcal{M}_7 \times S^3$ solution of type I supergravity corresponding to the fivebrane with shifted harmonic function must correspond to the $1/2$ supersymmetric \mathcal{M}_7 solution with linear dilaton of gauged $N = 2$ $d = 7$ supergravity [29] found in [30]. Also, the solution of a string in a five-brane gives rise, after the shift, to $adS_3 \times E^4 \times S^3$ geometry, and $adS_3 \times T^4$ is a solution of $N=2$ $d=7$ gauged supergravity as well [31]. These observations strongly suggest that $N=1$ $d=10$ supergravity compactified on a three-sphere yields $N=2$ $d=7$ gauged supergravity (see also [32]). Similar studies might be made for the other cases.

We have seen that dualities can connect spacetimes with different asymptotic geometries, leading to several interesting applications. A more detailed account of the issues reported here, and further extensions, will be discussed elsewhere [33].

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References

- [1] C.M. Hull and P.K. Townsend, *Nucl. Phys.* **B438** (1995) 109, hep-th/9410167; E. Witten, *Nucl. Phys.* **B443** (1995) 85, hep-th/9503124.
- [2] J.H. Schwarz, *Lectures on Superstrings and M theory dualities*, hep-th/9607201; P. Townsend, *Four Lectures on M theory*, hep-th/9612121.
- [3] T. Banks, W. Fischler, S.H. Shenker and L. Susskind, *Phys. Rev.* **D55** (1997) 5112, hep-th/9610043.
- [4] G. Horowitz and D. Welch, *Phys. Rev. Lett.* **71** (1993) 328, hep-th/9302126; E. Alvarez, L. Alvarez-Gaumé, J.L.F. Barbon, Y. Lozano, *Nucl. Phys.* **B415** (1994) 71, hep-th/9309039.
- [5] I. Bakas, *Phys. Lett.* **343B** (1995) 103, hep-th/9410104.
- [6] A. Dabholkar and J. Harvey, *Phys. Rev. Lett.* **63** (1989) 478; A. Dabholkar, G. Gibbons, J. Harvey and F. Ruiz Ruiz, *Nucl. Phys.* **B340** (1990) 33.

- [7] J. Breckenridge, G. Michaud and R.C. Myers, *More D-brane bound states*, hep-th/9611174; M. Costa and G. Papadopoulos, *Superstring dualities and p-brane bound states*, hep-th/9612204.
- [8] S. Hyun, *U-duality between Three and Higher Dimensional Black Holes*, hep-th/9704005.
- [9] M. Bañados, C. Teitelboim and J. Zanelli, *Phys. Rev. Lett.* **69** (1992) 1849, hep-th/9204099.
- [10] M. Cvetič and A. Tseytlin, *Nucl. Phys.* **B478** (1996) 431, hep-th/9606033.
- [11] G.T. Horowitz and A. Strominger, *Nucl. Phys.* **B360** (1991) 197.
- [12] R. Güven, *Phys. Lett.* **276B** (1992) 49.
- [13] G. Papadopoulos and P.K. Townsend, *Phys. Lett.* **380B** (1996) 273, hep-th/9603087; A. Tseytlin, *Nucl. Phys.* **B487** (1997) 141, hep-th/9609212; R. Argurio, F. Englert and L. Houart, *Intersection rules for p-branes*, hep-th/9701042.
- [14] E. Bergshoeff, M. de Roo, E. Eyras, B. Janssen and J.P. van der Schaar, *Multiple Intersections of D-branes and M-branes*, hep-th/9612095, and *Intersections involving monopoles and waves in eleven dimensions*, hep-th/9704120.
- [15] G.W. Gibbons and P.K. Townsend, *Phys. Rev. Lett.* **71** (1993) 3754, hep-th/9307049.
- [16] P.G.O. Freund and M.A. Rubin, *Phys. Lett.* **97B** (1980) 233.
- [17] R. Kallosh and J. Kumar, *Supersymmetry enhancement of D-p-branes and M-branes*, hep-th/9704189.
- [18] G.W. Gibbons, in *Supersymmetry, Supergravity and Related Topics*, eds. F. del Aguila, J. de Azcárraga and L. Ibáñez (World Scientific, Singapore, 1985), p. 147; R. Kallosh and A. Peet, *Phys. Rev.* **D46** (1992) 5223, hep-th/9209116; A. Chamseddine, S. Ferrara, G.W. Gibbons, R. Kallosh, *Enhancement of Supersymmetry Near 5d Black Hole Horizon*, hep-th/9610155.
- [19] H. Lü, C.N. Pope and P.K. Townsend, *Domain walls from anti-de Sitter spacetime*, hep-th/9607164.
- [20] O. Coussaert and M. Henneaux, *Phys. Rev. Lett.* **72** (1994) 183, hep-th/9310194.
- [21] E. Bergshoeff, R. Kallosh and T. Ortin, *Phys. Rev.* **D51** (1995) 3009, hep-th/9410230.

- [22] E. Alvarez, L. Alvarez-Gaumé and I. Bakas, *Nucl. Phys.* **B457** (1995) 3, hep-th/9507112.
- [23] M.J. Duff, S. Ferrara, R. Khuri and J. Rahmfeld, *Phys. Lett.* **356B** (1995) 479, hep-th/9506057.
- [24] M.J. Duff and J.X. Lu, *Nucl. Phys.* **B416** (1994) 301, hep-th/9306052.
- [25] L. Susskind, *Another Conjecture about M(atrix) theory*, hep-th/9704080.
- [26] I. Chepelev and A.A. Tseytlin, *Long-distance interactions of D-brane bound states and longitudinal 5-brane in M(atrix) theory*, hep-th/9704127; K. Becker, M. Becker, J. Polchinski and A. Tseytlin, *Higher order graviton scattering in M(atrix) theory*, hep-th/9706072; A. Tseytlin, *Talk given at Strings '97, Amsterdam*.
- [27] Some references are (and further references may be found in) B. de Wit and H. Nicolai, *Nucl. Phys.* **B281** (1987) 211; K. Pilch, P. van Nieuwenhuizen and P.K. Townsend, *Nucl. Phys.* **B242** (1984) 377; M. Günaydin, L.J. Romans and N.P. Warner, *Phys. Lett.* **B154** (1985) 268.
- [28] M. Duff, G. Gibbons and P. Townsend, *Phys. Lett.* **332B** (1994) 321, hep-th/9405124.
- [29] P. Townsend and P. van Nieuwenhuizen, *Phys. Lett.* **125B** (1983) 41; A. Salam and E. Sezgin, *Phys. Lett.* **126B** (1983) 295; E. Bergshoeff, I.G. Koh and E. Sezgin, *Phys. Rev.* **D32** (1985) 1353.
- [30] H. Lü, C.N. Pope and E. Sezgin, *Phys. Lett.* **371B** (1996) 46, hep-th/9511203.
- [31] S.K. Han, I.G. Koh and H.-W. Lee, *Phys. Rev.* **D32** (1985) 32.
- [32] M. Duff, P. Townsend and P. van Nieuwenhuizen, *Phys. Lett.* **122B** (1983) 232.
- [33] H.J. Boonstra, B. Peeters and K. Skenderis, *in preparation*.